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Primordial ^{4}He Abundance Constrains the Possible Time Variation of the Higgs Vacuum Expectation Value

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Abstract We constrain the possible time variation of the Higgs vacuum expectation value (v) by recent results on the primordial ^{4}He abundance (Y_P). For that, we use an analytic approach which enables us to take important issues into consideration, that have been ignored by previous works, like the v-dependence of the relevant cross sections of deuterium production and photodisintegration, including the full Klein–Nishina cross section. Furthermore, we take a non-equilibrium Ansatz for the freeze-out concentration of neutrons and protons and incorporate the latest results on the neutron decay. Finally, we approximate the key-parameters of the primordial ^{4}He production (the mean lifetime of the free neutron and the binding energy of the deuteron) by terms of $\frac{v}{v_0}$ (where v_0 denotes the present theoretical estimate). Eventually, we derive the relation $Y_P \simeq 0.2479 - 5.54(\frac{v-v_0}{v_0})^2 - 0.808(\frac{v-v_0}{v_0})$ and the most stringent limit on a possible time variation of v is given by: $-5.4 \times 10^{-4} \leq \frac{v-v_0}{v_0} \leq 4.4 \times 10^{-4}$.

1 Introduction

The standard model [9] is a remarkably successful description of fundamental particle interactions. The theory contains parameters—such as particle masses—whose origins are still unknown and which cannot be predicted, but whose values are constrained through their interactions with the so called Higgs field. The Higgs field is assumed to have a non-zero value in the ground state of the universe—called its vacuum expectation value v—and elementary particles that interact with the Higgs field obtain a mass proportional to this fundamental constant of nature.

Although the question whether the fundamental constants are in fact constant, has a long history of study (see [24] for a review), comparatively less interest [5, 11, 15, 22, 25] has been directed towards a possible variation of v.

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A macroscopic probe to determine the allowed variation range is given by the network of nuclear interactions during the Big-Bang-Nucleosynthesis (see [6] for a review of the Standard Big-Bang-Nucleosynthesis Model), with its final abundance of ^{4}He. The relevant key-parameters are the freeze-out concentration of neutrons and protons, the so called deuterium bottleneck (the effective start of the primordial nucleosynthesis) and the neutron decay.

The major difference between our contribution and previous studies is, that we further improve this key-parameters by using an analytic Ansatz, exclusively. This analytic approach enables us to take important issues into consideration, that have been ignored by previous works, like the v-dependence of the relevant cross sections of deuterium production and photodisintegration, including the full Klein–Nishina cross section. Furthermore, we take a non-equilibrium Ansatz for the freeze-out concentration of neutrons and protons and incorporate the latest results on the neutron decay.

Finally we approximate the mean lifetime of the free neutron and the binding energy of the deuteron by terms of v, to constrain its possible variation by recent results on the primordial ^{4}He abundance [3, 8, 12, 13, 16, 19].

We briefly note, that constraints on the spacial variation of v required a measurement of helium abundance anisotropy or inhomogeneity versus the position in the sky and an inhomogeneous theoretical BBN model. The homogeneous formalism used throughout the paper thus assumes a spacial invariance of the Higgs vacuum expectation value.

2 Calculations

All relevant processes of SBBN took place at a very early epoch, when the energy density was dominated by radiation, leading to a time-temperature relation for a flat universe:

$$t = \sqrt{\frac{90\hbar^3 c^5}{32\pi^3 k^4 G g_*}} \frac{1}{T^2}\ [\mathrm{s}], \tag{1}$$

where c is the velocity of light, k the Boltzmann constant, G denotes the gravitational constant and $\hbar$ is the Planck constant divided by 2π. g_* counts the total number of effectively massless ($mc^2 \ll kT$) degrees of freedom, given by $g_* = (g_b + \frac{7}{8} g_f)$, in which g_b represents the bosonic and g_f the fermionic contributions at the relevant temperature. The non-relativistic species are neglected, since their energy density is exponentially smaller [14].

At very high temperatures ($T \gg 10^{10}$ K), the neutrons and protons are kept in thermal and chemical equilibrium by the weak interactions

$$\begin{aligned} &n + e^+ \rightleftarrows p + \bar{\nu}_e, \\ &n + \nu_e \rightleftarrows p + e^- \text{ and} \\ &n \rightleftarrows p + e^- + \bar{\nu}_e, \end{aligned}$$

until the temperature drops to a certain level, at which the inverse reactions become inefficient. This so called "freeze-out"-temperature T_f and time t_f denote the start of the effective neutron beta decay and detailed calculations [18] derive

$$\left(\frac{kT_f}{Q}\right)^2 \left(\frac{kT_f}{Q} + 0.25\right)^2 \simeq 0.18 \sqrt{\frac{\pi^2}{30} g_*(T_f)}, \tag{2}$$

where Q is the energy-difference of the neutron and proton rest masses. At T_f the effectively massless species in the cosmic plasma are neutrinos, antineutrinos, electrons, positrons and photons. For the case of three neutrino families $g_b = 2$ and $g_f = 10$, which gives $g_*(T_f) = 10.75$.

Following Mukhanov [18] we take a non-equilibrium freeze-out ratio of neutron number density (n_n) to baryon number density (n_N):

$$\frac{n_n}{n_N}(T_f) = \int_0^\infty \frac{\exp[-5.42(\frac{\pi^2}{30} g_*(T_f))^{-\frac{1}{2}} \int_0^y (x+\frac{1}{4})^2 (1+e^{-\frac{1}{x}}) dx]}{2y^2(1+\cosh(1/y))} dy, \tag{3}$$

where $y = kT_f/Q$. From now on the decay of free neutrons via $n \to p + e^- + \bar{\nu}_e$, with a mean lifetime [23] $\tau_n = 878.5$ sec, can no longer be refreshed. Thus, whereas the neutron density decreases as $n_n(t) = n_n(t_f) \cdot e^{-\frac{t-t_f}{\tau_n}}$, the proton density increases as $n_p(t) = n_p(t_f) + (n_n(t_f) - n_n(t))$ and we obtain

$$\frac{n_p}{n_n}(t) = \frac{n_N}{n_n}(T_f) e^{\frac{t-t_f}{\tau_n}} - 1. \tag{4}$$

The next important step is the start of nucleosynthesis t_N, usually referred to as the "deuterium bottleneck". The delay between t_f and t_N is caused by the very low efficiency of direct production of light elements by successive collisions of several free protons and neutrons to one nucleus. In fact, nucleosynthesis proceeds through sequences of two-body reactions with the deuteron d as the intermediate product, via $p + n \to d + \gamma$. Accordingly, the small binding energy of the deuteron $B_d \simeq 2.225$ MeV presents a severe problem for nucleosynthesis, since energetic photons of the background radiation continuously disrupt the newly formed deuterons, until the temperature drops to a certain level T_N, when the deuteron production gets the upper hand over photodisintegration. Because the decaying neutrons can no longer be refreshed by weak interactions after t_f, the interval between t_f and t_N plays an essential role for the outcome of the primordial helium production.

Hence, we have to calculate the rates of deuteron production $\Gamma_{(np\to d\gamma)}$, deuteron photodisintegration $\Gamma_{(d\gamma\to np)}$ and the expansion rate of the universe $\Gamma_{\exp}$, to determine t_N respectively T_N, when

$$\frac{\Gamma_{(np\to d\gamma)}}{\Gamma_{(d\gamma\to np)}} > 1 + \frac{\Gamma_{\exp}}{\Gamma_{(d\gamma\to np)}}. \tag{5}$$

$\Gamma_{\exp}$ for a radiation-dominated, flat universe is given by $\frac{1}{2t}$ with t from (1). The rates for production and photo-disintegration of deuteron are given by the product of the relevant number density, velocity and cross section (σ):

$$\frac{\Gamma_{(np\to d\gamma)}}{\Gamma_{(d\gamma\to np)}} = \frac{n_p \sqrt{\frac{3kT}{m_N}} \sigma_{(np\to d\gamma)}}{n_\gamma^* c \sigma_{(d\gamma\to np)}} = \frac{\eta \sqrt{\frac{3kT}{m_N}} \sigma_{(np\to d\gamma)}}{(1+\frac{n_n}{n_p}(T)) \frac{n_\gamma^*}{n_\gamma} c \sigma_{(d\gamma\to np)}}, \tag{6}$$

where $\eta \simeq 6.14 \times 10^{-10}$ is the baryon to photon ratio based on WMAP [10] and n_γ^* denotes the number density of photons which effectively disintegrate the deuteron. These photons have to supply enough energy and must not loose this energy in much more likely Compton scattering with electrons instead of deuterons.

The number density of photons at a certain temperature T is given by

$$n_\gamma = \frac{8\pi}{(hc)^3} \int_0^\infty \frac{E^2}{e^{\frac{E}{kT}} - 1} dE = 16\pi \zeta(3) \left(\frac{kT}{hc}\right)^3, \tag{7}$$

where ζ is the Riemann zeta function. The number density of these photons, supplying a minimum energy $E_{\text{dis}} \gg kT$ is

$$n_{(\gamma > E_{\text{dis}})} = \frac{8\pi}{(hc)^3} \int_{E_{\text{dis}}}^{\infty} E^2 e^{-\frac{E}{kT}} dE = 8\pi \left(\frac{kT}{hc}\right)^3 \left[\left(\frac{E_{\text{dis}}}{kT} + 1\right)^2 + 1\right] e^{-\frac{E_{\text{dis}}}{kT}} \tag{8}$$

whereas only a fraction $\frac{\sigma_{(d\gamma \to np)}}{\frac{n_p}{n_n}(T)\sigma_{(e\gamma \to e\gamma)}}$ will successfully disintegrate a deuteron leading to

$$n_\gamma^* = n_{(\gamma > E_{\text{dis}})} \frac{\sigma_{(d\gamma \to np)}}{\frac{n_p}{n_n}(T)\sigma_{(e\gamma \to e\gamma)}}. \tag{9}$$

We take $\sigma_{(e\gamma \to e\gamma)} \simeq \sigma_{KN}(E_\gamma)$ where σ_{KN} denotes the Klein–Nishina cross section [21] for electron photon scattering and the mean incident photon energy E_γ is given by

$$E_\gamma = \frac{1}{n_{(\gamma > E_{\text{dis}})}} \frac{8\pi}{(hc)^3} \int_{E_{\text{dis}}}^{\infty} E^3 e^{-\frac{E}{kT}} dE \simeq E_{\text{dis}} + kT. \tag{10}$$

The interaction cross section of photo-disintegration $\sigma_{(d\gamma \to np)}$ can be derived by the calculations of Rustgi and Pandey [20]. We use a least mean square approximation within the incident photon energy range of 2.3 to 3.6 MeV and obtain

$$\sigma_{(d\gamma \to np)} = \frac{1}{n_{(\gamma > E_{\text{dis}})}} \frac{8\pi}{(hc)^3} \int_{E_{\text{dis}}}^{\infty} E^2 e^{-\frac{E}{kT}} [-2162.3 + 8.1208 \times 10^{15}(E + 1.5kT)] dE \tag{11}$$

$$\simeq -2162.3 + 8.1208 \times 10^{15}(E_{\text{dis}} + 2.5kT)\ [\mu\text{b}]. \tag{12}$$

The cross section for neutron capture $\sigma_{(np \to d\gamma)}$ is related to $\sigma_{(d\gamma \to np)}$ by [2]:

$$\frac{\sigma_{(np \to d\gamma)}}{\sigma_{(d\gamma \to np)}} \simeq \frac{3E_\gamma^2}{2m_N c^2 (E_\gamma - B_d)}, \tag{13}$$

where we take E_γ from (10). Collecting all terms, inserting into (5) and taking into account, that $E_{\text{dis}} = B_d - \frac{3}{2}kT$ we obtain an equation for T_N:

$$\left(\frac{\eta\zeta(3)\frac{n_p}{n_n}(T_N)}{1 + \frac{n_n}{n_p}(T_N)}\right) \sqrt{\frac{(3kT_N)^3}{m_n^3 c^6}} \frac{\sigma_{(e\gamma \to e\gamma)} e^{\frac{B_d}{kT_N} - \frac{3}{2}}}{\sigma_{(d\gamma \to np)}}$$

$$= 1 + \frac{n_p}{n_n}(T_N)\pi^2 \sqrt{\frac{Gg_*(T_N)h^3}{90c}} \frac{\sigma_{(e\gamma \to e\gamma)} kT_N e^{\frac{B_d}{kT_N} - \frac{3}{2}}}{\sigma_{(d\gamma \to np)}^2 10^{-34} (B_d - \frac{1}{2}kT_N)^2} \tag{14}$$

where

$$\frac{n_p}{n_n}(T_N) = \frac{n_N}{n_n}(T_f) \exp\left[\frac{1}{\tau_n} \sqrt{\frac{90\hbar^3 c^5}{32\pi^3 k^4 G}} \left(\frac{1}{\sqrt{g_*(T_N)}T_N^2} - \frac{1}{\sqrt{g_*(T_f)}T_f^2}\right)\right] - 1 \tag{15}$$

and $\frac{n_N}{n_n}(T_f)$ is the reciprocal freeze-out ratio, given by (3). The factor 10^{-34} on the right hand side of (14) is due to the unit μb that we use for all cross sections σ.

The neutrinos have decoupled from equilibrium at about one MeV (above the rest mass energy of an electron) and thus before the annihilation of electron positron pairs. Therefore the entropy due to this annihilation is transferred exclusively to the photons, i.e. $g_*(T_N) \simeq 3.36$.

Once sufficient deuteron has been produced, all other reactions

$$
\begin{aligned}
{}^2_1d + p &\rightleftarrows {}^3_2\mathrm{He}, \\
{}^2_1d + {}^2_1d &\rightleftarrows {}^4_2\mathrm{He}, \\
{}^3_2\mathrm{He} + {}^3_2\mathrm{He} &\rightleftarrows {}^4_2\mathrm{He} + 2p
\end{aligned}
$$

proceed with significantly higher binding energies and the nucleosynthesis is no longer constrained by photo-disintegration. It ends when the thermal energy is insufficient to permit the energetically favored synthesis reaction—the fusion of deuteron.

With the assumption that at t_N all available neutrons (as well as the same number of protons) have been synthesized to ^{4}He, which is not further transformed into heavier nuclei, because elements with nucleon mass number $A = 5$ to $A = 8$ are insufficiently stable to function successfully as intermediate products for nucleosynthesis at the available densities, we can calculate Y_P, the final ^{4}He abundance by weight:

$$
Y_P = \frac{2}{1 + \frac{n_p}{n_n}(t_N)} = \frac{2\frac{n_n}{n_N}(T_f)}{e^{\frac{t_N - t_f}{\tau_n}}} = \frac{2\frac{n_n}{n_N}(T_f)}{\exp\left[\frac{1}{\tau_n}\sqrt{\frac{90\hbar^3 c^5}{32\pi^3 k^4 G}}\left(\frac{1}{\sqrt{g_*(T_N)}T_N^2} - \frac{1}{\sqrt{g_*(T_f)}T_f^2}\right)\right]}, \tag{16}
$$

where $\frac{n_n}{n_N}(T_f) \simeq 0.15709$ is given by (3), $T_f \simeq \frac{Q}{k}\,0.64794$ is given by (2) and (14) determines T_N, respectively.

For comparison with the most recent numerical result [4] $Y_P^{\mathrm{num}} = 0.2483$, which assumes a mean neutron lifetime $\tau_n = 885.7$ [s], we obtain (see (16)): $Y_P(\tau_n = 885.7) = 0.2483$.

For comparison with the observation-based result [3] $Y_P^{\mathrm{obs}} = 0.2479$, we take [23] $\tau_n = 878.5$ [s] and obtain (see (16)): $Y_P(\tau_n = 878.5) = 0.2479$. Furthermore, this calculation shows, how sensitive Y_P depends on the mean lifetime of neutrons.

Taking into account our simple approach for the start of nucleosynthesis (see (5)), where we neglected the fact, that the deuterons are not only destroyed by photo-disintegration but also consumed by the fusion of light elements, the concordance with the numerical as well as the observation-based result is very encouraging.

This analytic expression for Y_P therefore provides our basis for finding the dependence of Y_P and the possible deviation of v from its present value v_0, in order to finally constrain $\frac{v}{v_0}$ by recent results on the primordial ^{4}He abundance.

Crucial for the result of primordial nucleosynthesis is the moment t_N or the corresponding temperature T_N, at which the production rate gets the upper hand. T_N depends on the binding energy of the deuteron B_d. Hence we take the linear fit of B_d versus m_π that has been used by Yoo and Scherrer [25] and Müller et al. [17], based on Beane and Savage [1]:

$$
B_d(v) \simeq B_d(v_0)\left(11 - 10\frac{m_\pi}{m_{\pi_0}}\right), \tag{17}
$$

where m_π is the pion mass (the index 0 again denotes the present value). As emphasized by Yoo and Scherrer [25], in our narrow range of interest, $m_\pi^2 \propto v$, leading to the final expression

$$
B_d(v) \simeq B_d(v_0)(11 - 10\sqrt{v/v_0}). \tag{18}
$$

As B_d changes, E_{dis}, E_γ and the cross sections $\sigma_{(d\gamma\to np)}$, $\sigma_{(np\to d\gamma)}$ and $\sigma_{(e\gamma\to e\gamma)}$ change, accordingly. Concerning $\sigma_{(e\gamma\to e\gamma)}$ we furthermore have to consider, that the mass of the electron varies proportionally

$$m_e(v) = m_e(v_0)\frac{v}{v_0} \tag{19}$$

which enters the Klein–Nishina cross section.

Any variation of v, of course, changes the value of Q as well, but according to (2), T_f is proportional to Q with the interesting consequence, that the freeze-out concentration of neutrons and protons does not change with a varying Q.

By contrast, the Higgs vacuum expectation value definitely influences the mean lifetime of the free neutrons τ_n and following [17] we use the expression

$$\begin{aligned}\frac{\tau_n - \tau_{n_0}}{\tau_{n_0}} &= 3.86\frac{\alpha-\alpha_0}{\alpha_0} + 4\frac{v-v_0}{v_0} + 1.52\frac{m_e - m_{e_0}}{m_{e_0}}\\ &\quad - 10.4\frac{(m_d - m_u) - (m_{d_0} - m_{u_0})}{(m_{d_0} - m_{u_0})},\end{aligned} \tag{20}$$

where α is the electromagnetic fine structure constant and m_d and m_u are the masses of the up- and down-quark (the index 0 again denotes the present values). This approximation is the result of a linear analysis, based on the assumption, that only one single fundamental coupling changes with time while keeping the others fixed, respectively. Furthermore, Müller et al. only consider standard model particles (with three neutrino families) to contribute to the energy density at BBN.

Taking into account, that the elementary masses of the electrons and quarks linearly depend on v and disregarding the effect of a varying α (we take α as constant throughout this letter[1]), we achieve

$$\tau_n(v) \simeq \tau_n(v_0)\left(1 - 4.88\frac{v-v_0}{v_0}\right). \tag{21}$$

Finally, we derive a relation between Y_P and v, to constrain the permitted variation of the Higgs vacuum expectation value by the primordial ^{4}He abundance:

$$Y_P \simeq 0.2479 - 5.54\left(\frac{v-v_0}{v_0}\right)^2 - 0.808\left(\frac{v-v_0}{v_0}\right). \tag{22}$$

The predominant effect is the variation of the binding energy of the deuteron, followed by the mean neutron lifetime, whereas the changing mass of the electron with its consequences on the Klein–Nishina cross section (and the Compton scattering respectively) is almost negligible. The relative weights can be quantified by the linearization

$$\begin{aligned}Y_P &\simeq 0.2479 + 0.105\left(\frac{B_d - B_{d_0}}{B_{d_0}}\right) + 0.058\left(\frac{\tau_n - \tau_{n_0}}{\tau_{n_0}}\right)\\ &\quad - 0.006\left(\frac{m_e - m_{e_0}}{m_{e_0}}\right).\end{aligned} \tag{23}$$

[1] We also take $\hbar$, k, c, G and η as constant throughout the letter.

Table 1 The permitted variation of v constrained by different observation based results on Y_p

Authors	Y_P	Permitted variation $\frac{(v-v_0)}{v_0}$
Fukugita and Kawasaki [8]	0.250 ± 0.004	$(-8.0\,^{+5.0}_{-5.3}) \times 10^{-3}$
Olive and Skillman [19]	0.2491 ± 0.0091	$(-1.5\,^{+10.6}_{-12.5}) \times 10^{-3}$
Coc et al. [3]	0.2479 ± 0.0004	$(-0.49\,^{+4.9}_{-4.9}) \times 10^{-4}$
Izotov et al. [12]	0.2443 ± 0.0015	$(4.3\,^{+1.7}_{-1.7}) \times 10^{-3}$
Izotov and Thuan [13]	0.2421 ± 0.0021	$(6.8\,^{+2.3}_{-2.3}) \times 10^{-3}$
Luridiana et al. [16]	0.2391 ± 0.0020	$(10.1\,^{+2.1}_{-2.2}) \times 10^{-3}$

Table 2 The Higgs vacuum expectation value constrained by different observation based results on Y_p

Authors	Compatible v_0 in GeV
Fukugita and Kawasaki [8]	$245.56\,^{+1.22}_{-1.31}$
Olive and Skillman [19]	$245.84\,^{+2.63}_{-3.08}$
Coc et al. [3]	$246.21\,^{+0.12}_{-0.12}$
Izotov et al. [12]	$247.28\,^{+0.42}_{-0.44}$
Izotov and Thuan [13]	$247.90\,^{+0.57}_{-0.59}$
Luridiana et al. [16]	$248.72\,^{+0.52}_{-0.54}$

3 Results

Using the observation based results of Coc et al. [3] we derive

$$-5.4 \times 10^{-4} \le \frac{v - v_0}{v_0} \le 4.4 \times 10^{-4}.$$

We avoid the term "observational results" because all cited publications more or less consist of interpretation of observational ^{4}He-abundance plus theoretical input and constraints by the cosmic microwave background. Especially the different interpretation as a result of the deficiently understood systematics lead to incompatible data. Therefore, we separately state all publications and their constraints on v in Table 1.

Taking $v_0 = v$ as constant, the primordial ^{4}He abundance can be used for another interesting subject. The Higgs vacuum expectation value can be calculated theoretically by [5]

$$v_0 = 2^{-1/4} G_f^{-1/2} \simeq 246.22 \text{ [GeV]}, \tag{24}$$

where $G_f \simeq 1.166371$ [GeV^{-2}] is the Fermi coupling constant [7], based on measurements of the muon mass and lifetime. The uncertainty of determining v_0, especially the contributions of higher order terms, can now be constrained in Table 2.

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References

1. Beane, S.R., Savage, M.J.: Nucl. Phys. A **717**, 91 (2003)
2. Chen, J.-W., Savage, M.J.: Phys. Rev. C **60**, 065205 (1999)
3. Coc, A., Vangioni-Flam, E., Descouvemont, P., Adahchour, A., Angulo, C.: Astrophys. J. **600**, 544 (2004)
4. Cuoco, A., et al.: Int. J. Mod. Phys. A **19**, 4431 (2004)
5. Dixit, V.V., Sher, M.: Phys. Rev. D **37**, 1097 (1988)
6. Eidelman, S., et al.: Phys. Lett. B **592**, 1 (2004)
7. Eidelman, S., et al.: Phys. Lett. B **592**, 1 (2004) and 2005 partial update for the 2006 edition available on PDG www pages (URL: http://pdg.lbl.gov/)
8. Fukugita, M., Kawasaki, M.: Astrophys. J. **646**, 691 (2006)
9. Griffiths, D.: Introduction to Elementary Particles. Wiley, Canada (1987)
10. Huey, G., Cyburt, R.H., Wandelt, B.D.: Phys. Rev. D **69**, 103503 (2004)
11. Ichikawa, K., Kawasaki, M.: Phys. Rev. D **65**, 123511 (2002)
12. Izotov, Y.I., et al.: Astrophys. J. **527**, 757 (1999)
13. Izotov, Y.I., Thuan, T.X.: Astrophys. J. **602**, 200 (2004)
14. Kolb, E.W., Turner, M.S.: The Early Universe. Westview (1990)
15. Kujat, J., Scherrer, R.J.: Phys. Rev. D **62**, 023510 (2000)
16. Luridiana, V., Peimbert, A., Peimbert, M., Cerviño, M.: Astrophys. J. **592**, 846 (2003)
17. Müller, Ch.M., Schäfer, G., Wetterich, Ch.: Phys. Rev. D **70**, 083504 (2004)
18. Mukhanov, V.: Int. J. Theor. Phys. **43**, 669 (2004)
19. Olive, K.A., Skillman, E.: Astrophys. J. **617**, 29 (2004)
20. Rustgi, M.L., Pandey, L.N.: Phys. Rev. C **40**, 1581 (1989)
21. Rybicki, G.B., Lightman, A.P.: Radiative Processes in Astrophysics. Wiley, Canada (1979)
22. Scherrer, R.J., Spergel, D.N.: Phys. Rev. D **47**, 4774 (1993)
23. Serebrov, A., Varlamov, V., Kharitonov, A., Fomin, A., Pokotilovski, Yu., Geltenbort, P., Butterworth, J., Krasnoschekova, I., Lasakov, M., Taldaev, R., Vassiljev, A., Zherebtsov, O.: Phys. Lett. B **605**, 72–78 (2005)
24. Uzan, J.P.: Rev. Mod. Phys. **75**, 403 (2003)
25. Yoo, J.J., Scherrer, R.J.: Phys. Rev. D **67**, 043517 (2003)